P318 Computational Modeling
Spring 2014
Week 8 Homework
Due: Monday March 10th

This assignment will ask you to implement in Matlab some of the Monte Carlo methods we discussed in class.

Each of the following exercises will ask you to use the Beta distribution. In class, we talked about random number generators for distributions like the uniform, normal, and exponential. We also talked about creating random number generators for arbitrary distributions. The Beta distribution is just one possible distribution out there in the universe of probability distributions. This one happens to be a widely used probability in Bayesian statistical analyses, a topic we will be discussing next.

Its probability density function is defined for values of x in the range [0, 1], and has the following mathematical form

$$p(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}$$

where $B(\alpha,\beta)$ is the Beta function, which is defined as

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt$$

Fortunately, the Beta function is predefined in Matlab, so you do not need to calculate this integral explicitly. But you'll need to find the right function for yourself in the Matlab documentation. Note: Do not confuse the Beta distribution with the Beta function. Also don't confuse either with the β parameter. They are all different things, despite the similarity in their names. The Beta distribution is a probability distribution function, the Beta function is a function that's defined by an integral (but is not a probability distribution), and β is a parameter used in those two.

For this assignment, when you print out or plot solutions to the exercises in your Matlab code, assume that α=2 and β=3, but your code should be written so that it accepts any value of α or β greater than or equal to one (values of α or β less than one are allowed with the Beta distribution, but the coding can get a bit more harry, which is why you can limit it to positive values). Note that I may check some other values of α and β when I grade your assignment, so make sure it's easy for me to find where to change these parameter values in your Matlab code (and that I only need to change alpha and beta in one place, not multiple places).

As you work in the assignment, please make sure you write readable code. Also, make sure your plots are fully labeled and properly formatted; of course, all plots should be generated in Matlab code.

1. First, plot the probability density function for the Beta distribution given particular values of α or β. First do this by plotting the function for the Beta distribution above. Next, do this by plotting the function using the build-in Matlab function that returns the Beta distribution (the beta distribution pdf, not the random number generator for the Beta distribution – remember that those are different).

2. Look in the Matlab documentation to find the built-in random number generator for the Beta distribution. Generate 1000 random samples from the Beta distribution. Create a histogram from these samples. Plot the histogram. It should look a little like the plot of the probability distribution you created for part 1 (you might confirm that for yourself by running it with 10x or 100x the number samples, but I only want to run your code for 1000 when I test your code).

3. Now create your own random number generator for the Beta distribution using the rejection sampling method we discussed in class. Generate 1000 random samples, create a histogram, and a plot, just as you did for part 2. Remember that you'll need to make sure that your code works for any combination of α or β greater than or equal to one.