PSY318
Week 4
course web site
Note: For all homework assignments, when graphs and plots are needed, I want them to be created by Matlab code and to be properly formatted and labeled. On the course web site is a simple example of some Matlab code for creating line and bar graphs: GraphExamples.m.

For more information on creating graphs and plots in Matlab, you might check my PSY319 course (especially Week 6): http://catlab.psy.vanderbilt.edu/palmeri/psy319/
Another note on homework assignments:

Students in the class for credit can talk with each other about solving various coding issues and help each other with roadblocks. Just make sure the bulk of the assignment is done by yourself.

For people sitting in, I can’t stress enough how important it is to at least try the assignments - at a minimum sketching out a solution, or trying some coding yourself, then reviewing the solution I post after the assignment is turned in.
in-depth with similarity choice model, MDS-choice model, and generalized context model

mathematically simple, relatively easy to implement

use these models to illustrated concepts like model parameterization, model fitting, hierarchical model testing
Categorization
what kind of object is this?

Recognition
have I seen this object before?

Identification
what is this one’s name?
modeling identification
each stimulus $i$ is assigned a unique name $j$.

subject is tasked with learning those names
Imagine we are tasked with developing a model of identification.

At some point we will ask the question: Can the same representations and processes support both the identification and the categorization of objects?

But for now, let’s just worry about creating a model.

Training
- see stimulus $i$
- give response $j$
- get corrective feedback

Testing
- see stimulus $i$
- give response $j$
First:
What are we going to model?
P(R_j|S_i) is the probability of naming stimulus i with response j (identification confusions)
$P(R_j|S_i)$ is the probability of naming stimulus $i$ with response $j$ (identification confusions)

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$16 \times 16 = 256$ data points
freq(R_j|S_i) is the frequency of naming stimulus \( i \) with response \( j \) (identification confusions)
### Table 1

**Condition AS Confusion Data**

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<td>24</td>
<td>95</td>
<td>0</td>
<td>7</td>
<td>106</td>
<td>170</td>
<td>0</td>
<td>6</td>
<td>64</td>
<td>50</td>
</tr>
<tr>
<td>13 (1, 4)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>21</td>
<td>6</td>
<td>0</td>
<td>0</td>
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<td>57</td>
<td>3</td>
<td>0</td>
<td>239</td>
<td>99</td>
<td>5</td>
<td>0</td>
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<tr>
<td>14 (2, 4)</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>16</td>
<td>9</td>
<td>3</td>
<td>31</td>
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<td>48</td>
<td>11</td>
<td>34</td>
<td>205</td>
<td>103</td>
<td>20</td>
</tr>
<tr>
<td>15 (3, 4)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>7</td>
<td>15</td>
<td>0</td>
<td>25</td>
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<td>95</td>
<td>2</td>
<td>36</td>
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</tr>
<tr>
<td>16 (4, 4)</td>
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<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>16</td>
<td>1</td>
<td>3</td>
<td>54</td>
<td>106</td>
<td>0</td>
<td>3</td>
<td>93</td>
<td>308</td>
</tr>
</tbody>
</table>

*Note. Rows correspond to stimuli and columns correspond to responses.

* Angle, Size

**Nosofsky (1986)**
Each stimulus ($S_i$) is assigned a unique name ($R_j$)
let's develop a model of object identification

**FIG. 14.2.** Illustration of stimulus set used in Nosofsky’s (1985b) experiment.
\[ P(R_j | i) = \frac{E_j}{\sum_{K \in R} E_K} \]

starting point is Luce's choice model
$P(R_j \mid i) = \frac{E_j}{\sum_{K \in R} E_K}$

$E_j$ is a free parameter specifying the evidence that stimulus $i$ should get label $j$
\[ P(R_j \mid i) = \frac{E_j}{\sum_{K \in R} E_K} \]

why are we dividing?
\( P(R_j | i) = \frac{\beta_j E_j}{\sum_{K \in R} \beta_K E_K} \)

bias \((\beta)\) is a free parameter
\[ P(R_j \mid i) = \frac{\beta_j E_j}{\sum_{K \in R} \beta_K E_K} \]

bias (\(\beta\)) is a free parameter

we don’t have a theory of how to set bias, so its value can be anything ...

we will attempt to optimize the value of bias

or we can assume equal biases (unbiased responses)
\[ P(R_j | i) = \frac{\beta_j E_j}{\sum_{K \in R} \beta_k E_k} \]

this equation could be used irrespective of the model of identification (computer vision model, similarity, signal detection theory)

what are the evidences \((E_j)\)
\[ P(R_j | i) = \frac{\beta_j E_j}{\sum_{K \in R} \beta_K E_K} \]

bias (\( \beta \)) is a free parameter

why is bias multiplied by evidence?

what does this do mathematically?
similarity-choice model (SCM) of identification

\[ P(R_j | i) = \frac{\beta_j \cdot s_{ij}}{\sum_{K \in R} \beta_K \cdot s_{iK}} \]


similarity-choice model (SCM) of identification

\[ P(R_j|i) \] is the probability of naming stimulus \( i \) with the label for stimulus \( j \) (identification confusions)
similarity-choice model (SCM) of identification

\[ P(R_j \mid i) = \frac{\beta_j \cdot s_{ij}}{\sum_{K \in R} \beta_K \cdot s_{iK}} \]

ASSUMPTIONS:

\( 0 \leq s_{ij} \leq 1 \)

\( s_{ii} = 1 \)

some theories haven’t assumed this (e.g., Krumhansl)
similarity-choice model (SCM) of identification

\[ P(R_j \mid i) = \frac{\beta_j \cdot s_{ij}}{\sum_{K \in R} \beta_K \cdot s_{iK}} \]

\[ P(R_i \mid i) = \frac{\beta_j \cdot 1}{\sum_{K \in R} \beta_K \cdot s_{iK}} \]
similarity-choice model (SCM) of identification

\[ P(R_j \mid i) = \frac{\sum_{K \in R} \beta_j \cdot s_{ij}}{\sum_{K \in R} \beta_K \cdot s_{iK}} \]

ASSUMPTIONS:

\[ 0 \leq s_{ij} \leq 1 \]

\[ s_{ii} = 1 \]

\[ s_{ij} = s_{ji} \]

an assumption challenged by Tversky
similarity-choice model (SCM) of identification

\[
P(R_j \mid i) = \frac{\beta_j \cdot s_{ij}}{\sum_{K \in R} \beta_K \cdot s_{iK}}
\]

\[
\beta = \begin{bmatrix}
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix} \quad s = \begin{bmatrix}
s_{11} & s_{12} & s_{13} \\
s_{21} & s_{22} & s_{23} \\
s_{31} & s_{32} & s_{33}
\end{bmatrix}
\]

e.g., with only 3 objects
similarity-choice model (SCM) of identification

\[ P(R_j \mid i) = \frac{\beta_j \cdot s_{ij}}{\sum_{K \in R} \beta_K \cdot s_{iK}} \]

\[ \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ 1 - \beta_1 - \beta_2 \end{bmatrix} \quad \mathbf{s} = \begin{bmatrix} 1 & s_{21} & s_{31} \\ s_{21} & 1 & s_{32} \\ s_{31} & s_{32} & 1 \end{bmatrix} \]

e.g., with only 3 objects
similarity-choice model (SCM) of identification

\[
P(R_j \mid i) = \frac{\beta_j \cdot s_{ij}}{\sum_{K \in R} \beta_K \cdot s_{iK}}
\]

\[
\beta = \begin{bmatrix}
\beta_1 \\
\beta_2 \\
1 - \beta_1 - \beta_2
\end{bmatrix} \quad \text{and} \quad s = \begin{bmatrix}
1 & s_{21} & s_{31} \\
s_{21} & 1 & s_{32} \\
s_{31} & s_{32} & 1
\end{bmatrix}
\]

e.g., with only 3 objects
FIRST HOMEWORK ASSIGNMENT
(1) implement similarity-choice model
(2) generate predictions based on parameter values

let’s talk about how to do this
\[ P(R_j \mid i) = \frac{\beta_j \cdot S_{ij}}{\sum_{K \in R} \beta_K \cdot S_{iK}} \]

what do you need to do to implement this in Matlab?
\[ P(R_j \mid i) = \frac{\prod_j \beta_j \cdot S_{ij}}{\sum_{K \in R} \prod_K \beta_K \cdot S_{iK}} \]

important programming pointers:
- comment your code
- use meaningful variable names
- use arrays/matrices/vectors when necessary
- use modular coding, creating functions when necessary
- use meaningful function names
- never use global variable (well, almost never)
models have free parameters
model development attempts to reduce the number of free parameters
model development attempts to reduce the number of free parameters

Explain the previously unexplained.

Turn a parameter into a formalized part of the model.
how might we reduce the number of free parameters?

\[
P(R_j \mid i) = \frac{\beta_j \cdot s_{ij}}{\sum_{K \in R} \beta_K \cdot s_{iK}}
\]
MDS-choice model of identification

\[
P(R_j \mid i) = \frac{\beta_j \cdot s_{ij}}{\sum_{K \in R} \beta_K \cdot s_{iK}}
\]

develop a theory of similarity
MDS-choice model of identification

\[ P(R_j \mid i) = \frac{\beta_j \cdot s_{ij}}{\sum_{K \in R} \beta_K \cdot s_{iK}} \]

Where do the \( s_{ij} \) parameters come from?

Can we use a theory of similarity rather than assume that similarity is a free parameter?

\( s_{ij} = \ ??? \)
**templates** – similarity a function of shape overlap

**dimensions** – objects are represented as points in a multidimensional psychological space, with similarity a function of distance in that space (Shepard)

**features** – objects are represented in terms of parts that can be present or absent, with similarity a function of feature overlap (Tversky)
if objects are represented as points in a multidimensional psychological space, then similarity is an inverse function of distance in that space ...
\[ d_{ij} = \sqrt{(i_1 - j_1)^2 + (i_2 - j_2)^2} \]

Euclidian distance metric
\[
\begin{align*}
\mathbf{d}_{ij} &= |i_1 - j_1| + |i_2 - j_2| \\
\text{“city-block” distance metric}
\end{align*}
\]

separable-dimension stimuli

vs.

integral-dimension stimuli
The general distance metric is given by:

\[ d_{ij} = \sum_{m=1}^{M} \left( (i_m - j_m)^r \right)^{1/r} \]

for a general metric, where:

- \( r = 1 \) if city-block
- \( r = 2 \) if Euclidean
similarity is a decreasing function of distance

*what function?*
similarity is a decreasing function of distance

**what function?**

\[
S_{ij} = \frac{1}{d_{ij}}
\]

\[
S_{ij} = \frac{1}{1 + d_{ij}}
\]

\[
S_{ij} = e^{-d_{ij}} = \exp(-d_{ij})
\]

\[
S_{ij} = e^{-d_{ij}^2} = \exp(-d_{ij}^2)
\]
similarity is a decreasing function of distance

*what function?*

\[ S_{ij} = \frac{1}{d_{ij}} \]

\[ S_{ij} = \frac{1}{1 + d_{ij}} \]

\[ S_{ij} = e^{-d_{ij}} = \exp(-d_{ij}) \]

\[ S_{ij} = e^{-d_{ij}^2} = \exp(-d_{ij}^2) \]

*you could just try a bunch and see “which works”*
similarity is a decreasing function of distance
what function?

\[ s_{ij} = \frac{1}{d_{ij}} \]

\[ s_{ij} = \frac{1}{1 + d_{ij}} \]

\[ s_{ij} = e^{-d_{ij}} = \exp(-d_{ij}) \]

\[ s_{ij} = e^{-d_{ij}^2} = \exp(-d_{ij}^2) \]

maybe there's a theoretical justification
for one over another
\[ g(d) = 1 - \frac{d}{\mu} + \frac{d}{\mu} \log \frac{d}{\mu}, \quad d \leq 2\mu \]

\[ g(d) = 1 - \left(\frac{2d}{3\mu}\right)^2, \quad d \leq \frac{3}{2}\mu \]

\[ g(d) = 1 - \left(\frac{d}{3\mu}\right)^2 + 2\left(\frac{d}{3\mu}\right) \log \frac{d}{3\mu}, \quad d \leq 3\mu \]

\[ g(d) = 1 - \frac{d}{2\mu} + 3\left(\frac{d}{2\mu}\right)^2 - \left(\frac{d}{2\mu}\right)^3, \quad d \leq 2\mu \]

\[ g(d) = \exp\left(-\frac{d}{\mu}\right) - \frac{d}{\mu} \operatorname{Ei}\left(\frac{d}{\mu}\right), \quad d \leq \mu \]

\[ g(d) = \exp\left(-2\frac{d}{\mu}\right) \]
similarity is a decreasing function of distance 
what function?

\[ S_{ij} = \frac{1}{d_{ij}} \]

\[ S_{ij} = \frac{1}{1 + d_{ij}} \]

\[ *S_{ij} = e^{-d_{ij}} = \exp(-d_{ij}) \]

\[ S_{ij} = e^{-d_{ij}^2} = \exp(-d_{ij}^2) \]

* Toward a Universal Law of Generalization (Shepard, 1987)

rational basis for choosing this function
independent empirical evidence for this function too
similarity is a decreasing function of distance
what function?

$$S_{ij} = \frac{1}{d_{ij}}$$

$$S_{ij} = \frac{1}{1 + d_{ij}}$$

$$S_{ij} = e^{-d_{ij}} = \exp(-d_{ij})$$

$$**S_{ij} = e^{-d_{ij}^2} = \exp(-d_{ij}^2)$$

** exponential generalization PLUS Gaussian noise
similarity is a decreasing function of distance
what function?

\[ S_{ij} = e^{-c \cdot d_{ij}^p} = \exp(-c \cdot d_{ij}^p) \]

\( p \) specifies similarity function
\( p=1 \) exponential
\( p=2 \) Gaussian

c is the “sensitivity parameter”
generalization becomes steeper as \( c \) increases

differences in \( c \) can explain brain damage
(Nosofsky & Zaki, 1998) or
expertise (Palmeri et al., 2004; Mack et al., 2007)
FIRST HOMEWORK ASSIGNMENT
(1) implement similarity-choice model
(2) generate predictions based on parameter values
(3) implement MDS-choice model
(4) generate predictions based on parameter values
model development attempts to reduce the number of free parameters

Explain the previously unexplained.

Turn a parameter into a formalized part of the model.
$P(R_j|S_i)$ is the probability of naming stimulus $i$ with response $j$ (identification confusions)

16 x 16 = 256 data points
256-16=240 free data points (why?)
16x15=240 free data points

Similarity-choice model:

\[
P(R_j \mid i) = \frac{\beta_j \cdot S_{ij}}{\sum_{K \in R} \beta_K \cdot S_{iK}}
\]

how many free bias parameters?

how many free similarity parameters?
16x15 = 240 free data points

Similarity-choice model:

\[ P(R_j | i) = \frac{\beta_j \cdot S_{ij}}{\sum_{K \in R} \beta_K \cdot S_{iK}} \]

16-1 = 15 free bias parameters

16x(16-1)/2 = 120 free similarity parameters
16\times 15 = 240 \text{ free data points}

\text{Similarity-choice model:} \\
16 - 1 = 15 \text{ free bias parameters} \\
16 \times (16 - 1) / 2 = 120 \text{ free similarity parameters} \\
135 \text{ free parameters total}

\text{MDS-choice model}

\[ S_{ij} = e^{-c \cdot d_{ij}^p} = \exp(-c \cdot d_{ij}^p) \]

How many free parameters?
16x15 = 240 free data points

Similarity-choice model:
16-1 = 15 free bias parameters
16x(16-1)/2 = 120 free similarity parameters
135 free parameters total

MDS-choice model

\[ S_{ij} = e^{-c \cdot d_{ij}^p} = \exp(-c \cdot d_{ij}^p) \]

16-1 = 15 free bias parameters
16 stimuli x 2 dimensions = 32 location parameters (30 free)
1 c parameter, c=1 without loss of generality
1 p parameter, 1 r parameter
48 free parameters total
16x15=240 free data points

Similarity-choice model:
16-1=15 free bias parameters
16x(16-1)/2= 120 free similarity parameters
135 free parameters total

MDS-choice model

\[ S_{ij} = e^{-c \cdot d_{ij}^p} = \exp(-c \cdot d_{ij}^p) \]

How many free parameters?
16x15=240 free data points

Similarity-choice model:
16-1=15 free bias parameters
16x(16-1)/2= 120 free similarity parameters
135 free parameters total

MDS-choice model

\[ S_{ij} = e^{-c \cdot d_{ij}^p} = \exp(-c \cdot d_{ij}^p) \]

16-1=15 free bias parameters
4 dimension 1 locations (3 free)
4 dimension 2 locations (3 free)
1 c parameter (set c=1)
1 p parameter, 1 parameter
23 free parameters total
16x15 = 240 free data points

Similarity-choice model:
16 - 1 = 15 free bias parameters
16x(16 - 1)/2 = 120 free similarity parameters
135 free parameters total

MDS-choice model

\[ S_{ij} = e^{-c \cdot d_{ij}^p} = \exp(-c \cdot d_{ij}^p) \]

16 - 1 = 15 free bias parameters
location derived from MDS or psychophysics
1 c parameter (needed here for scaling)
1 p parameter, 1 r parameter
18 free parameters
16x15=240 free data points

Similarity-choice model:
16-1=15 free bias parameters
16x(16-1)/2= 120 free similarity parameters
135 free parameters total

MDS-choice model

\[ S_{ij} = e^{-c \cdot d_{ij}^p} = \exp(-c \cdot d_{ij}^p) \]

equal bias
location derived from MDS or psychophysics
1 c parameter
1 p parameter, 1 r parameter
3 free parameters total
16x15 = 240 free data points

Similarity-choice model:
16 - 1 = 15 free bias parameters
16 \times (16 - 1) / 2 = 120 free similarity parameters
135 free parameters total

MDS-choice model

\[ S_{ij} = e^{-c \cdot d_{ij}^p} = \exp(-c \cdot d_{ij}^p) \]

equal bias
location derived from MDS or psychophysics
1 c parameter
p = 1, r = 1 (based on previous work)
1 free parameter total
modeling categorization
\[ P(R_j \mid i) = \frac{\beta_j E_j}{\sum_{K \in R} \beta_k E_K} \]
\[ P(A | i) = \frac{\beta_A \cdot E_{A|i}}{\beta_A \cdot E_{A|i} + \beta_B \cdot E_{B|i}} \]

What are some ways categories could be represented?

What gives rise to the evidence values?
Prototypes

$E_{A|i}$ is proportional to similarity to the prototype of category A.
$E_{A|i}$ is proportional to similarity to the ideal point of category A.
$E_{A|i}$ is proportional to similarity to the experienced exemplars of category A
$E_{A|i}$ is given by which side of the boundary exemplar $i$ is on (boundary can be noise)
$E_{A|i}$ is given by which side of the rule boundary exemplar $i$ is on (boundary can be noise)
$E_{A|i}$ is proportional to similarity to the experienced exemplars of category A
$E_{A|i}$ is proportional to similarity to the experienced exemplars of category A.

- similarity to closest exemplar (nearest neighbor)
- average similarity to exemplars
- summed similarity to exemplars

\[ E_{A|i} = \sum_{j \in A} S_{ij} \]

\[ E_{A|i} = \sum_{j=1}^{N_A} S_{ij} \]